9:00 - 11: 00
32-155

MIT ID# (last four digits) SOLUTIONS

Unified Quiz FTM1

September 26, 2007

M - PORTION

EXAM SCORING:

#1M (50%)	
#2M (50%)	
FINAL SCORE	

Unified Quiz FTM1 - Materials and Structures Portion
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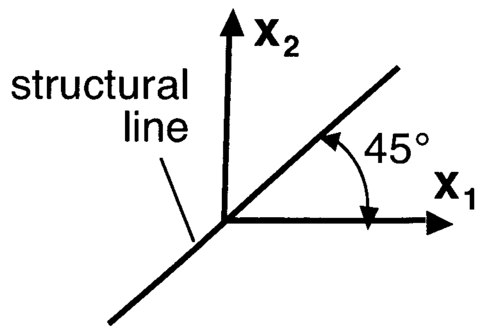
PROBLEM #1M (50%)

A set of four forces acts in the x_1-x_2 plane at points along a structural line through the origin at a 45° angle to the axis system. This configuration is illustrated in the accompanying figure. The force vectors and the (x_1, x_2) points at which they act are as follows:

$\underline{F}_1 = (2\text{ N})\,\underline{i}_1 + (5\text{ N})\,\underline{i}_2$	acts at (1m, 1m)
$\underline{F}_2 = (5\text{ N})\,\underline{i}_1$	acts at (-3m, -3m)
$\underline{F}_3 = (-3\text{ N})\,\underline{i}_1 + (2\text{ N})\,\underline{i}_2$	acts at (-1m, -1m)
$\underline{F}_4 = (-10\text{ N})\,\underline{i}_2$	acts at (5m, 5m)



(a) Determine the force system acting at the origin that is equipollent to this force system.

The total force and moment with regard to the origin must be determined in finding the equipollent force system acting at the origin

Force: must have same magnitude and direction or effect of these four forces.

Find: $\Sigma \underline{F}_i = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 + \underline{F}_4$

$+\uparrow \xrightarrow{+}$ $= (2N)\underline{i}_1 + (5N)\underline{i}_1 - (3N)\underline{i}_1 + (5N)\underline{i}_2 + (2N)\underline{i}_2 - (10N)\underline{i}_2$

$$\Rightarrow \boxed{\underline{F}_{(0,0)} = (4N)\underline{i}_1 + (-3N)\underline{i}_2}$$

Moment: Must include pure moment equal to sum of moments caused by four forces acting about origin:

$$\Sigma M_{(0,0)} = \Sigma m_i = \Sigma(\underline{r}_i \times \underline{F}_i)$$

~~Cross~~ product accounts for relative angle. To calculate, can take x_1 ~~or~~ x_2 component of each force and multiply by x_2 or x_1 distance to origin.

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PROBLEM #1M (continued)

Finally, note direction: ↺+

so: $\sum M_{(0,0)} = \{-(2N)(1m) + (5N)(1m) + (5N)(3m) - (3N)(1m) - (2N)(1m) - (10N)(5m)$

so: $M_{(0,0)} = -37 N\cdot m$ ↺+

summarizing: $F_{(0,0)} = (4N)\underline{i}_1 + (-3N)\underline{i}_2$
$M_{(0,0)} = -37 N\cdot m$

(b) Can this system be put in equilibrium by applying one force at any point along the structural line? If so, what is that force and what is the location? If not, explain why not and how many forces are needed and how these forces and their location can be determined. Clearly explain your reasoning.

One force can have the same magnitude and opposite direction for equilibrium. This is:

$$\underline{F} = (-4N)\underline{i}_1 + (3N)\underline{i}_2$$

This would need to be put at a point where it would cause exactly the opposite moment:

$$M = +37 N\cdot m \quad ↺+$$

The position vector is such that $x_1 = x_2$. So:

$$+37 N\cdot m = M = \{(+4N)(x_2) + (3N)(x_1)\}$$

setting $x_1 = x_2 = A$ gives:

$$37 N\cdot m = \{+7N\} A$$

$$\Rightarrow A = \frac{37}{7} m = x_1 = x_2$$

Answer is **YES**

Force is: $F = (-4N)\underline{i}_1 + (3N)\underline{i}_2$

Location is: $(37/7\, m, 37/7\, m)$

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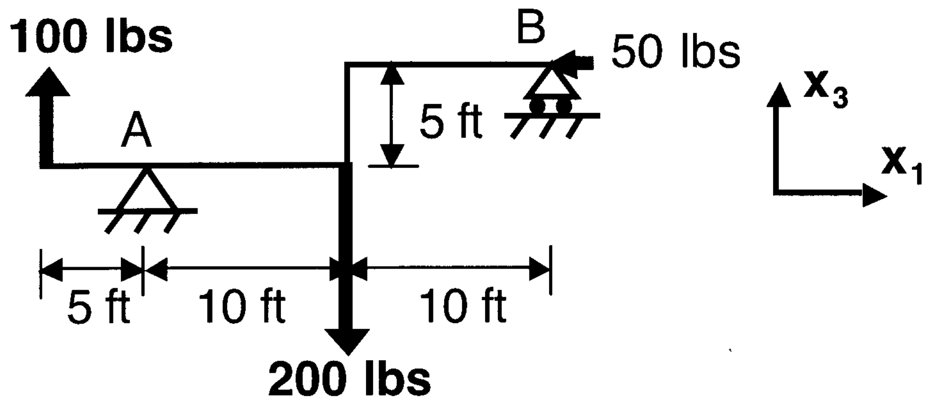
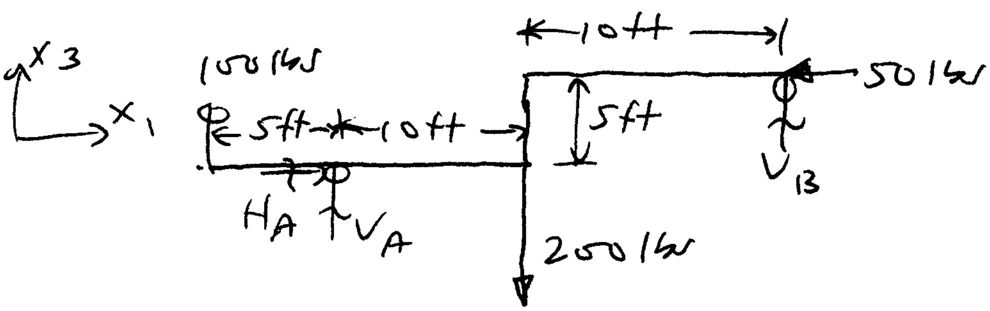
PROBLEM #2M (50%)

The 25-foot structure is supported along two height levels. At one side, there is a pin that the structure hangs beyond by 5 feet, while at the other end, a roller supports the structure at a level 5 feet above the pin. Loads are applied at three points: a 50-pound horizontal load at the roller support, a 200 pound vertical load at the midpoint between the supports, and a 100-pound vertical load at the far tip of the structure beyond the pin. This overall configuration is shown in the figure below.



(a) Determine the reaction forces at the two support points A and B ***or*** indicate all information available to determine the reaction forces and the additional information that is needed to fully determine these forces.

First draw the Free Body Diagram



Have 3 reactions and 3 degrees of freedom so this is statically determinate and can determine the reactions.

Use equations of equilibrium:

$\sum F_1 = 0 \quad \xrightarrow{+} \quad H_A - 50 \text{ lbs} = 0 \quad \Rightarrow \boxed{H_A = 50 \text{ lbs}}$

$\sum F_3 = 0 \quad \uparrow + \quad 100 \text{ lbs} + V_A - 200 \text{ lbs} + V_B = 0$

$\Rightarrow V_A + V_B = +100 \text{ lbs.}$

PROBLEM #2M (continued)

$$\sum M_{(\text{abt } A)} = 0 \quad \circlearrowleft + \quad -(100\text{ lbs})(5\text{ ft}) - (200\text{ lbs})(10\text{ ft}) + (50\text{ lbs})(5\text{ ft}) + V_B(20\text{ ft}) = 0$$

$$\Rightarrow -500\text{ ft lbs} - 2000\text{ ft lbs} + 250\text{ ft lbs} + V_B(20\text{ ft}) = 0$$

$$\Rightarrow \boxed{V_B = 112.5\text{ lbs}}$$

using $\sum F_3 = 0 \Rightarrow V_A = 100\text{ lbs} - V_B \Rightarrow \boxed{V_A = -12.5\text{ lbs.}}$

Summarizing:

$V_A = -12.5\text{ lbs}$
$V_B = 112.5\text{ lbs}$
$H_A = 50\text{ lbs}$

(b) How are these answers affected by the material from which the structure is made? Explain clearly.

The system is statically determinate and thus the reactions can be determined solely from the consideration of equilibrium.

Thus, the material **does not affect** the results